GRID FREEDOM GROUP
Your private utility provider

Reg number: 2017 / 052805 / 07

HEAT PUMPS
How they work

2 November 2017

Before analyzing heat pumps a brief recap of physics is required:

1. Properties of Fluids

1.1. Static Fluids

$$Density\ (\rho) = \frac{Mass}{Volume}$$

$$Pressure(Pascals) = \frac{Force\ (Newtons)}{Area}$$

$$\frac{dp}{dy} = pg$$

Or, the change in pressure on a submerged body is directly proportional to the distance below the surface (y), the liquid density (ρ) and gravity (g).

Pascals Principle: the same change in pressure applied to any point in a liquid at rest is transmitted to every part of the fluid.

$$p = p_0 + pgy$$

Where *pgy* is the weight per unit area.

This principle applies to hydraulic jacks where pressure applied to a small area is transmitted to a larger area connected to the load.

Archimedes Principle states that the buoyant force on an immersed object equals the weight of the displaced liquid.

This principle describes why boats float.

1.2. Fluids in Motion

Definitions:

Streamlines are small elements of liquid in motion
Laminar Flow is a steady flow without turbulence (Irrotational flow)
Turbulent flow has rotational flow patterns of eddies and vortices.

Assumptions:
- The fluid is incompressible
- The temperature does not vary
- Flow is steady
- Laminar flow only
- There is no viscosity

The equation of continuity (below):

$$v_1 A_1 = v_2 A_2 = flux\ \phi$$

Thus, describing the water exiting a garden hose nozzle.

Bernoulli's Equation applies Newton's Second Law to each element in the fluid to obtain:

$$p + \frac{1}{2} p v^2 + pgh = constant$$

The Bernoulli Effect describes the lift generated by an Aircraft's wing and the operation of a Venturi flowmeter. Pressure reduces as the velocity increases for all streamlines.

2. Temperature and Ideal Gasses

Boyles Law states that, for a dilute gas held at constant temperature, the product of pressure, p and volume V, divided by the number of moles, n, is constant:

$$\frac{pV}{n} = constant\ (for\ constant\ T)$$

The Ideal Gas Law:

$$pV = nRT$$

Number of moles = n
Where p is pressure, V is volume, n is number of moles, R is the universal gas constant and T is the temperature.

One mole (abbreviated mol) is the amount of gas in grams equal to the atomic weight of the gas. One mole of gas always contains Avogadro's Number of molecules = 6.022 x 10^{23}.

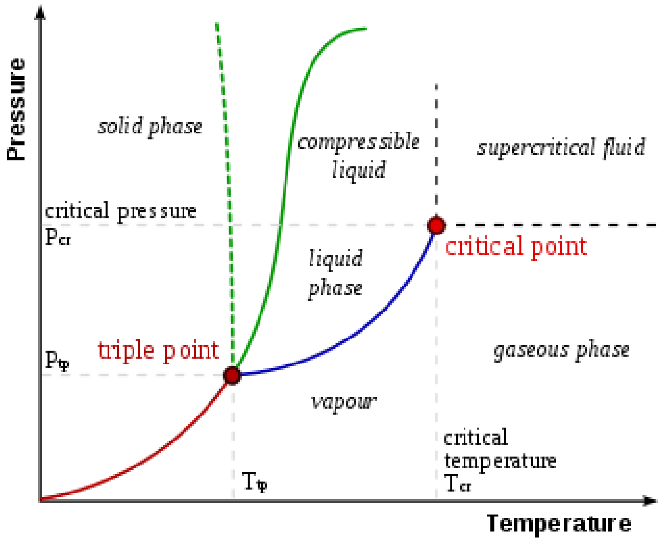
$$R = 8.314\frac{J}{mol}.K$$

If we count the number of molecules of a gas, N, rather than the number of moles, n, we get:

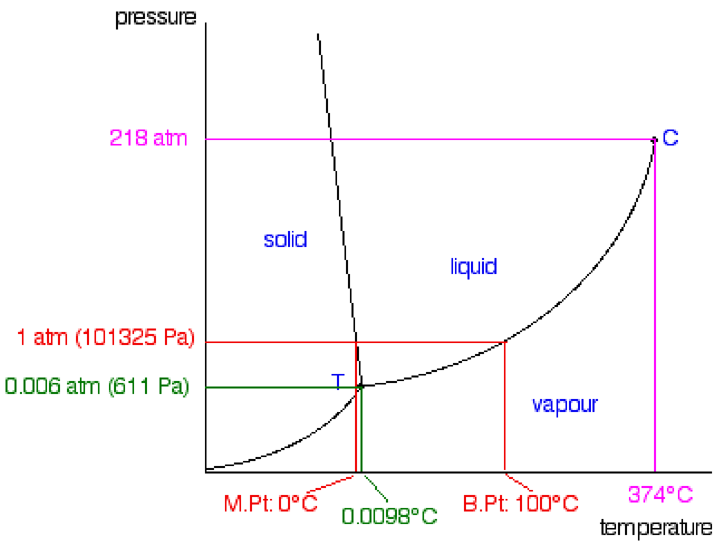
$$pV = NkT$$

Where k is Boltzmann's constant = 1.381 x 10^{-23} J/K

Changing thermodynamic variables of gas:
Isothermal: the temperature remains fixed whilst pressure and volume change.
Isobaric: the pressure is held fixed whilst the temperature and volume change.



The Triple point of water: a point where water can exist in gas, liquid and solid.



The van der Waals Equation of State (applicable to very dilute gases like air):

$$\left[p + a\left(\frac{n}{V}\right)^2\right]\left(\frac{V}{n} - b\right) = RT$$

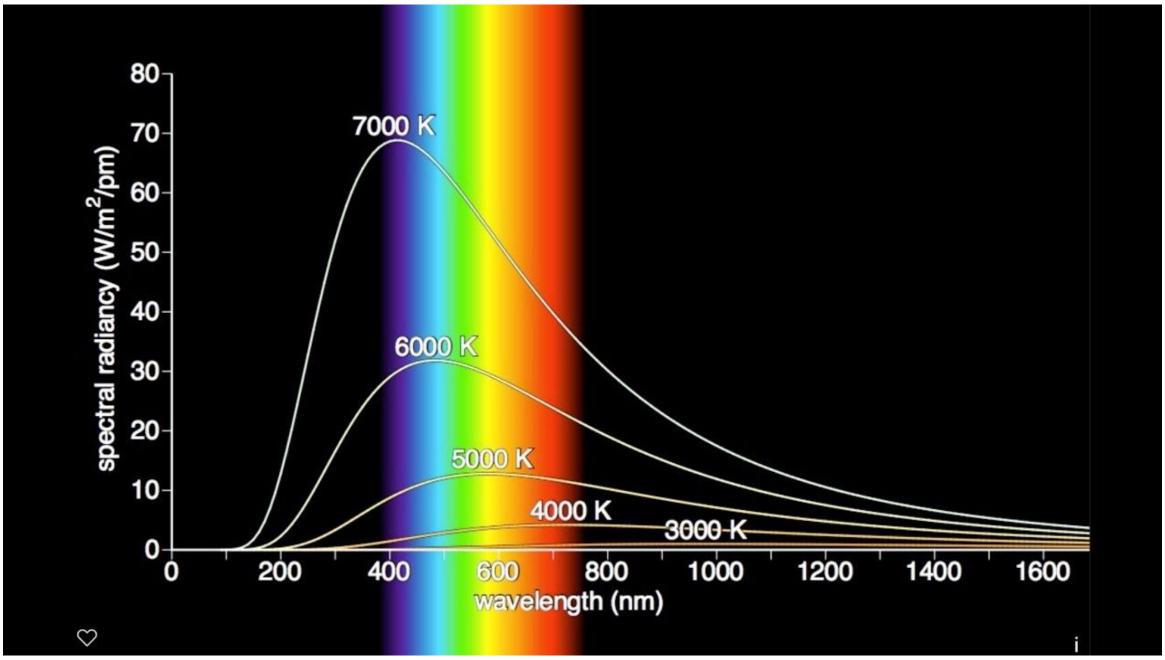
Black Body Radiation is the function of electromagnetic radiation and temperature.

The Planck Formula:

$$u(f,T) = \frac{8\pi h}{c^3} \cdot \frac{f^3}{e^{\frac{hf}{kT}} - 1}$$

Planck's Constant, h, = 6.625 x 10^{-34} J.s,
c = speed of light = 3 x 10^{8} ms^{-1}

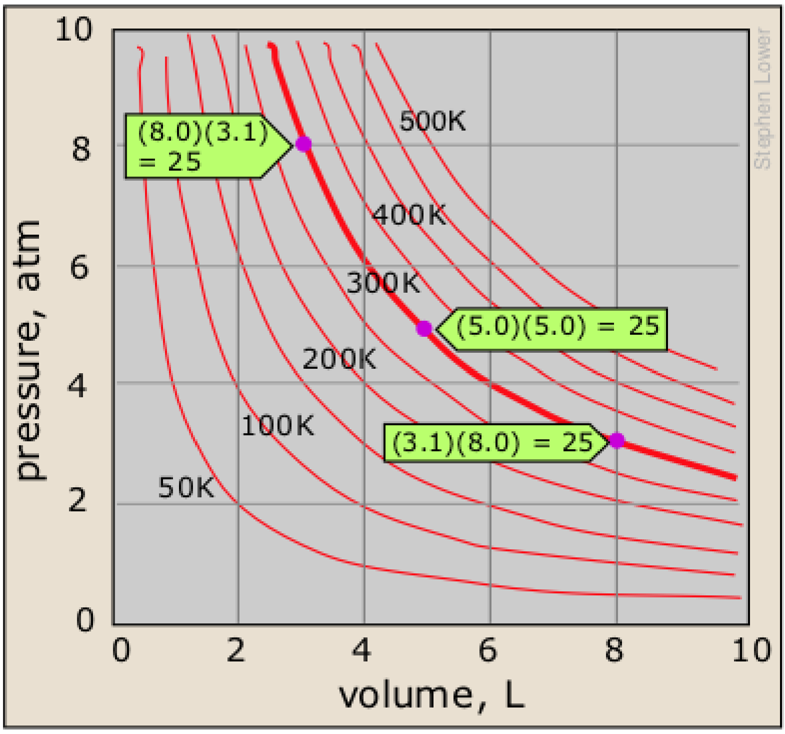
Black body radiation and colour temperatures are shown below:



The background temperature of the universe is measured as 3K. Absolute zero (0K) is -273.15^{0} C

3. First Law of Thermodynamics (Heat Flow)

Isothermal (an ideal gas being squashed and released in a cylinder) gas graphs are shown below:



The calorie (cal) is defined as the amount of heat flow required to raise the temperature of water of 1g of water at 1 atm from 14.5 to 15.5^{0}C.

The Btu (British thermal unit) is defined by the heat flow required to raise 1 pound of water by 1^{0}F.

1 Btu = 252.02 cal

3.1. Heat Capacity

This is the quantitative connection between heat flow and temperature, dQ:

$$dQ = CdT$$

Thus the heat flow equals the heat capacity over time.

Specific heat capacity, c, is the heat capacity of 1g of the substance.
Molar heat capacity is the heat capacity of 1 mol of the substance.

Substance	c/J kg^{-1} K^{-1}	Substance	c/J kg^{-1} K^{-1}
Aluminium	900	Ice	2100
Iron/steel	450	Wood	1700
Copper	390	Nylon	1700
Brass	380	Rubber	1700
Zinc	380	Marble	880
Silver	230	Concrete	850
Mercury	140	Granite	840
Tungsten	135	Sand	800
Platinum	130	Glass	670
Lead	130	Carbon	500
Hydrogen	14000	Ethanol	2400
Air	718	Paraffin	2100
Nitrogen	1040	Water	4186
Steam	2000	Sea water	3900

3.2. Path dependence and heat flow.

There are two special cases of interest. Either the volume or the pressure is kept constant. For constant volume:

$$dQ = C_V dT$$

And for constant pressure:

$$dQ = C_p dT$$

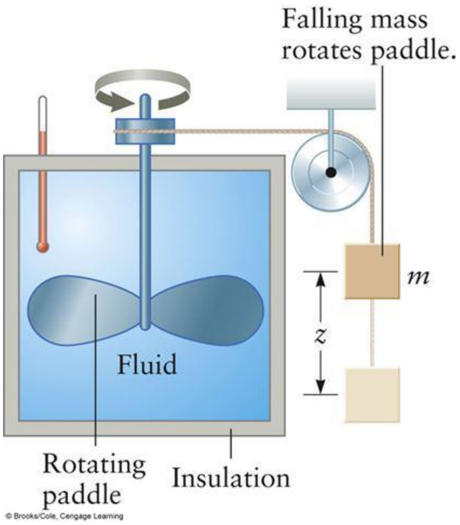
3.3. Phase changes and heat flow

When there is change of phase in a system there is no temperature change even though there is a heat flow. For example, it takes a certain heat flow to convert 1g of ice at its melting point of 0^0C to water at 0^0C. The necessary heat flow is called the **latent heat of fusion L_f.** L_f = 79.6 cal/g for ice. It also takes a certain amount of heat flow to convert 1g of water at its vaporization temperature 100^0C at 1 atm pressure, to 1g of steam. The heat flow necessary is called the **latent heat of vaporization.** This is 540 cal/g for water.

3.4. Mechanical equivalent of heat

Joule's heat flow experiment proved that change in temperature equals work done in an isolated system. This is known as the **mechanical equivalent of heat**:

$$\Delta Q = W$$



1 calorie = 4.185 Joule

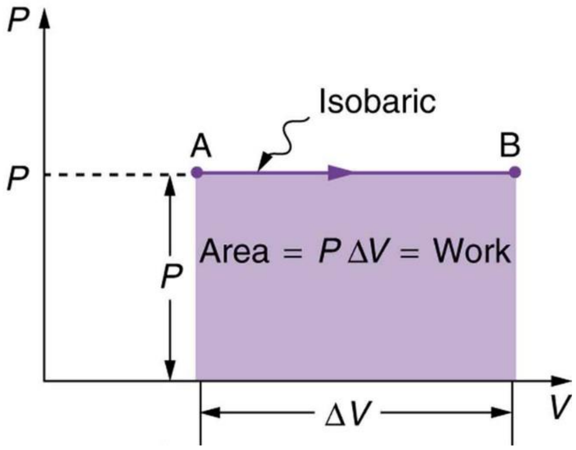
3.5. Work done in various transformations

Constant pressure: Generally, if the volume of the gas is to increase without a change in pressure then the temperature must increase. Using the ideal gas law, it follows that for a constant p:

$$dT = \frac{T}{V} dV$$

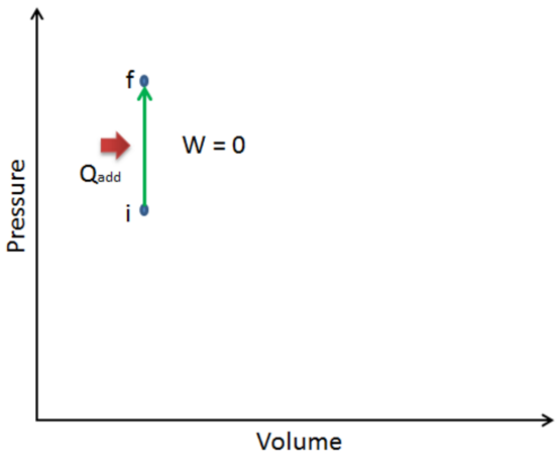
Hence the heat flow will be:

$$dQ = C_p dT$$

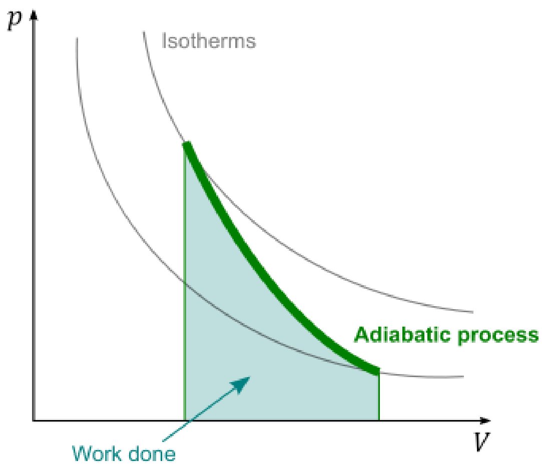


Constant volume: When the volume is kept fixed the curve of transformation is said to be **isochore.** Thermal systems do no work, positive or negative. In this case the heat flow is given by:

$$dQ = C_V dT$$



Adiabatic: There is a curve that is followed if the gas does work while it is thermally isolated from its environment. Imagine a cylinder in a thermal insulator with the piston connected to the environment. There is no dQ in or out of the system. When dQ = 0 the temperature can only change if work is done on the system. Reversible transformations of a thermal system in which there is no heat flow to the system are called adiabatic transformations. *Diesel engines are adiabatic, they squash the air so that it heats up dramatically causing spontaneous ignition of hydrocarbon fuels*.



The first law of thermodynamics is a statement of conservation of energy:

$$\Delta \boldsymbol{U} = \boldsymbol{U}_{\boldsymbol{B}} - \boldsymbol{U}_{\boldsymbol{A}} = -\boldsymbol{W}_{\boldsymbol{A}\to\boldsymbol{B}} + \boldsymbol{Q}_{\boldsymbol{A}\to\boldsymbol{B}}$$

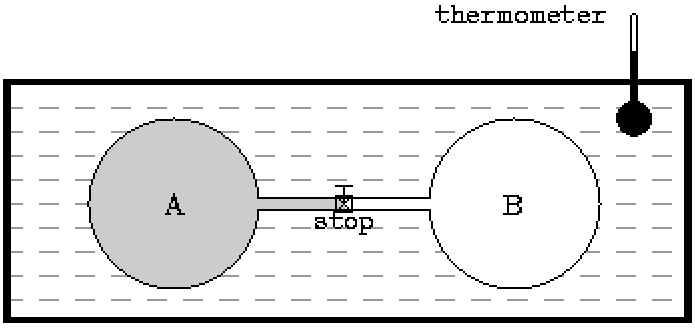
Hence, change in internal energy (U) equals the work done by the thermal system (negative means work going out) the heat flow into the system (Q). Or:

$$dU = -dW + dQ$$

The first law of thermodynamics in closed cycles:

$$Q_{cycle} = W_{cycle}$$

Also, for an ideal gas (sufficiently diluted), Joule showed that **the temperature of an ideal gas undergoing expansion remains constant**.



As internal energy is independent of volume then its energy is a function of temperature only:

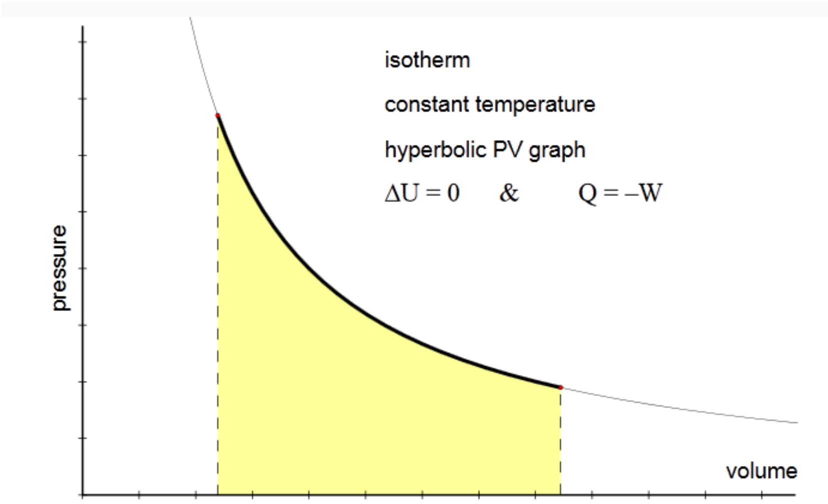
$$U_{ideal} = U(T)$$

For ideal gases:

$$C_p = C_V + nR$$

Isothermal: When the temperature is constant the work done by an ideal gas is simple to compute – it is simply the area under the curve. The integral of 1/x is the natural logarithm ln(x). So the work done by an ideal gas is given by:

$$W = nRT_0 ln\left(\frac{V_2}{V_1}\right)$$



Adiabatic Transformations of an ideal gas

The adiabatic curves that describe this transformation is given by:

$$pV^{\gamma} = P_0 V_0 = a\ constant$$

Variation of atmospheric temperature with height.

Adiabatic transformations explain the variation of atmospheric temperature with height. Air is a sufficiently good insulator so that the transport between different altitudes is adiabatic. As it rises its pressure decreases and so its temperature falls, so the air undergoes adiabatic expansion. The opposite occurs as it is carried to lower altitudes – it undergoes adiabatic compression and its temperature rises.

4. The second law of thermodynamics

Not all thermal energy in a thermal system is available to do work.
Thermal systems spontaneously change only in certain ways. Heat always flows from a hotter body to a colder one.

All engines have efficiency which is a measure of how well the heat flow is converted into work:

$$\eta = \frac{W}{Q_h}$$

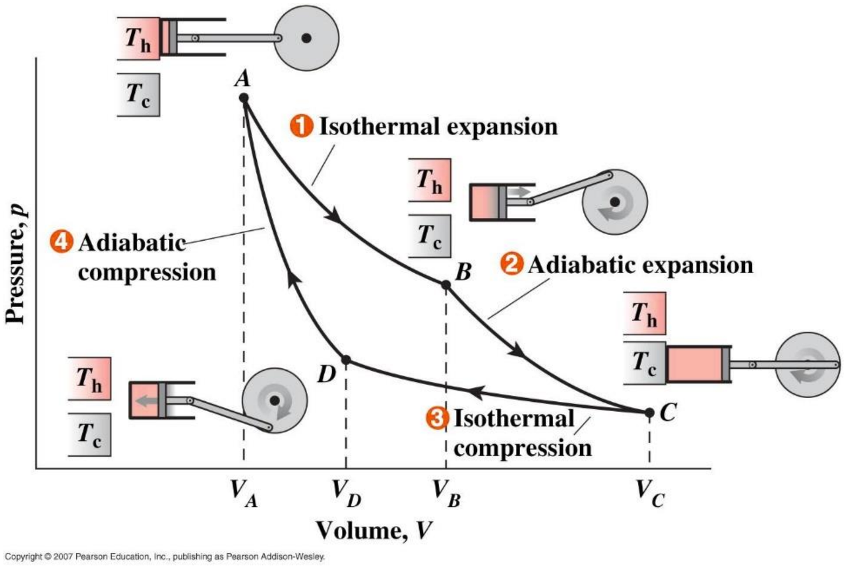
According to the first law of thermodynamics efficiencies can range from 0 to 1, although efficiencies of 100% are impossible.

4.1. The Carnot Cycle

Step 1 is isothermal. The expansion takes place with the cylinder in contact with a thermal reservoir (say burning petrol) at temperature T_H.

Step 2 is adiabatic expansion. The volume expands further whilst the pressure and temperature of the gas decrease. During this process the cylinder has been insulated from its surroundings so there is no heat flow – $\Delta Q = 0$. When the temperature has dropped to T_C it is placed in contact with a cold reservoir.

Step 3 is isothermal compression at temperature T_C. The final pressure of this step is determined so that when the thermal sink is removed and again placed in thermal isolation the adiabatic compression Step 4 increases the temperature and pressure of the gas to begin Step 1.



In the above diagram the isothermal lines are hyperbolas, and the adiabatic legs follow the curve:

$$p = (constant)V^{-\gamma}$$

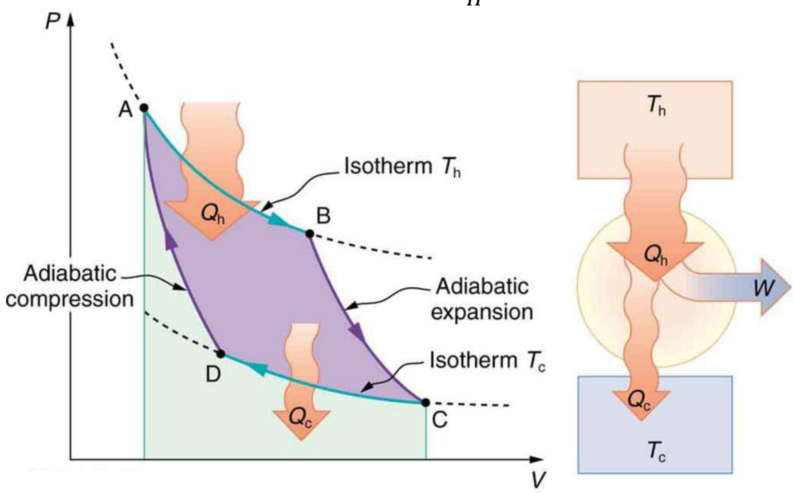
The work done in a Carnot Cycle is the area enclosed by the P-V diagram. A Carnot cycle therefore absorbs heat during step 1 and gives up heat during step 3.

The second law of thermodynamics implies two important results:

- All Carnot cycles that operate between the same two temperatures have the same efficiency. A Carnot cycle does not rely upon the use of an ideal gas.
- The Carnot engine is the most efficient possible that operates between the two given temperatures.

The efficiency of any Carnot cycle is given by:

$$\eta_C = 1 - \frac{T_C}{T_H}$$



5. Heat Pumps and Refrigerators

Heat pumps and fridges are nothing more than a heat engine in reverse. They transfer thermal energy from colder to hotter thermal reservoirs. Additional energy is required to perform work on the thermal system.

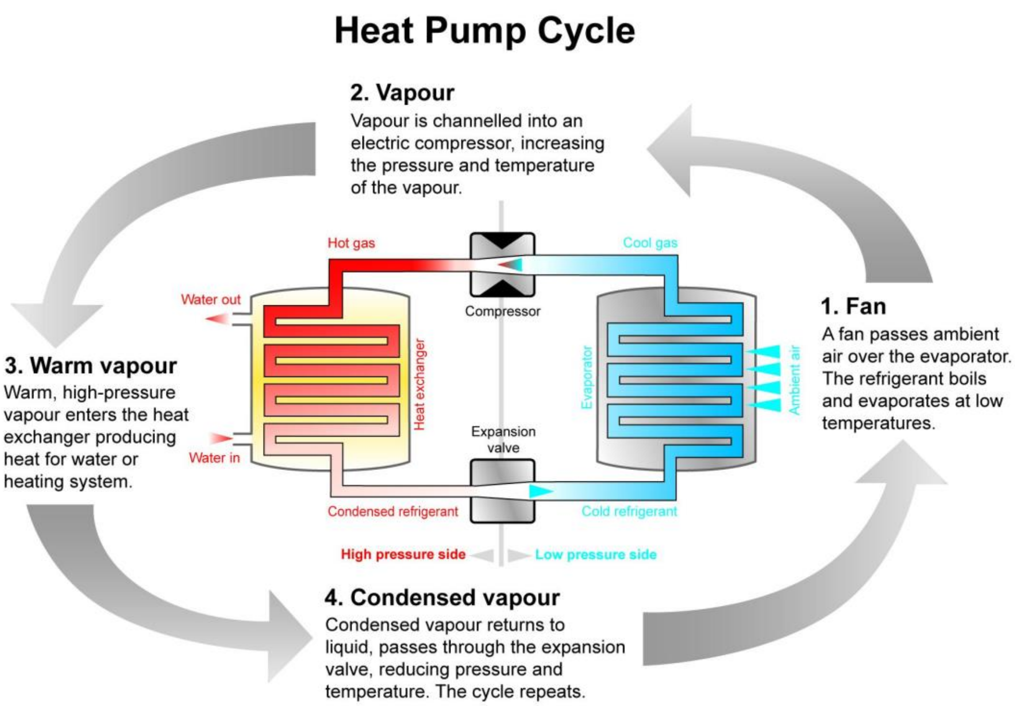
For a heat pump the total positive heat flow rejected from the pump into the building Q_{rej} is of interest. We use the coefficient of performance K_{hp}, which is defined by:

$$K_{hp} = \frac{Q_{rej}}{W}$$

Where W is the total work done in a cycle. It can be shown that:

$$K_{hp} = \frac{T_H}{T_H - T_C}$$

The coefficient of performance of a heat pump is greater than 1. It improves when the temperature differences are reduced. The ideal heat pump for heating a house in winter could operate between $T_C = -5^0C$ and $T_H = 45^0C$.



Coefficient of Performance

There are two types of devices, energy converters and energy transferring devices. The efficiency of a heat (energy) transferring devices is called the coefficient of performance (COP) unlike the energy conversion devices. COP is also the ratio of Energy Output to the Energy Input like the energy efficiency.

In an energy converter, the output will be a portion of the energy input and it may be less than the energy input. Therefore, the efficiency will be less than 100% by the laws of thermodynamics. In an energy transfer device, the energy output is the amount of heat extracted from the heat source (Space to be cooled-in case of refrigeration). The extracted energy is not a portion of the input energy. The extracted energy can exceed the input energy. Therefore, the efficiency of an energy transferring devices can be higher than 100% without violating the first law of thermodynamics. Therefore, the name coefficient of performance.

A heat pump is always more efficient than electric resistive heat. If you get a heat pump with resistive backup heat you wind up not saving a lot of money. I think a heat pump with natural gas backup is the best way to save money year round.

Heat pumps are between 3 to 5 times more efficient than electric heat over their useful range (65 to 30 degrees outside). Newton's Laws are not violated in a refrigeration cycle. In this cycle heat is not generated, it's moved. Heat generated with electric heating elements are close to 100% efficient, but when compared to a heat pump you get 3 to 5 times less heating capacity for the amount of energy you put in with resistive element heating when compared to a heat pump. Or another way, where a resistive element is 100% efficient, a heat pump is 300% to 500% as efficient trying to do the same function. You don't get to create heat energy with a heat pump, you do "work" with that energy to move the energy inside/outside. And it does this by making the outdoor coils even colder than it is outside in heating mode.

1. Take the datasheet of a heat pump:
http://www.goodmanmfg.com/Portals/0/pdf/SS/SS-SSZ14.pdf
2. Look at page 18.
3. See the COP (coefficient of power)
4. Notice how it is from 3.01 to 4.44 between 30 to 65 degrees outside.
5. That means that for the power coming in, you move the input power times that number worth of power.