COSINE DEVELOPMENTS

LEADERS IN LIGHTING TECHNOLOGY

Reg. No. IT 1637/97

Explaining
Distortion Power Factor (DPF)

By: Pete Pomeroy and Stirling Marais

The proliferation of electronic ballasts into our local market has been met with both awe and confusion. Yes, a good electronic ballast will bestow marvellous advantages over their electromagnetic rivals but how does one quantify the weird mains current waveforms? Whereas the power factor measurement for electromagnetic ballasts is simply a measure of displacement between the voltage and current fundamentals (both sinusoids) the effect of the transient current pulse of electronic ballasts requires further understanding.

Now it so happens that the analysis of the waveforms for electronic ballasts is nothing new: Its full wave rectifier front end has been used for decades in most power supplies. What is new is the sheer quantity of these things connected to our mains, especially with the new drive to eradicate incandescent lamps around the home.

What is power factor? Displacement power factor, distortion power factor..

Figure 1 shows the measured voltage and current waveforms for a typical off-the-mains full wave rectifier. The lower trace (current) is a current transient to top up the smoothing capacitor and typically leads the voltage slightly (by about 10^0). The pulse width is typically around 1 to 3 ms. The displacement power factor of this waveform is approximately 0.98 because the current is almost in phase with the voltage.

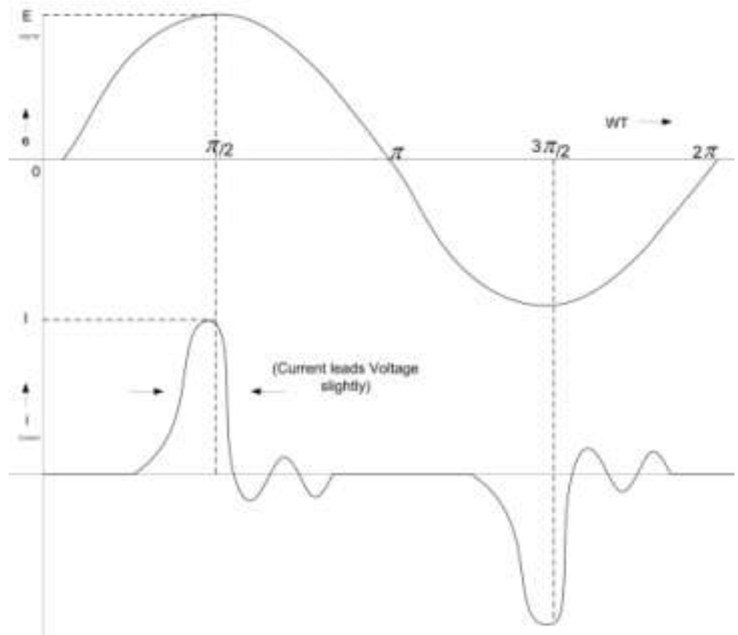

Figure 1

Our piecewise approximation of the current pulse is shown in Figure 2, its height being I and its width θ. Mean instantaneous power calculation is simply:

$$p = e.i$$

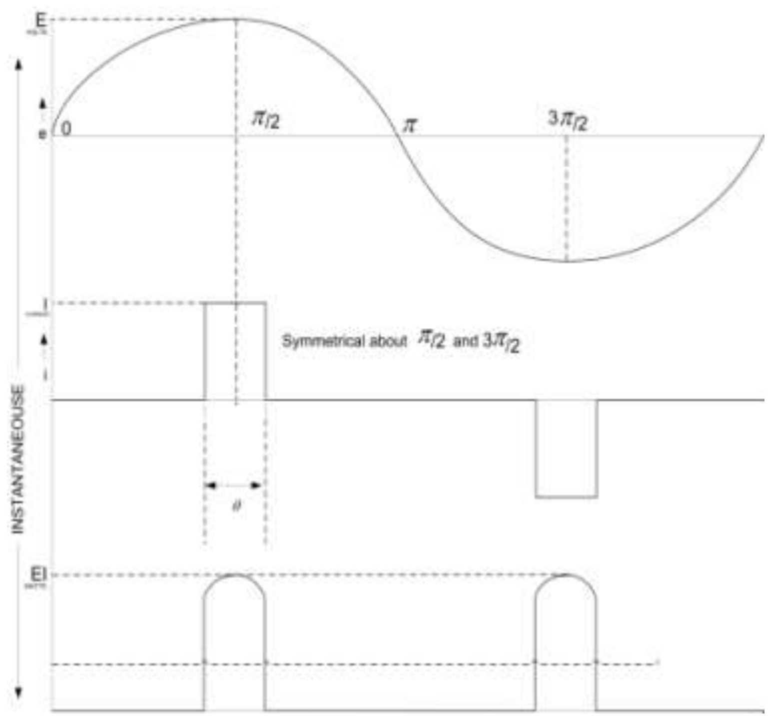

Figure 2

The mean power for a half cycle will be the instantaneous power averaged over a half cycle:

$$Mean\ power = \frac{1}{\pi}\int_{(\pi/2-\theta/2)}^{(\pi/2+\theta/2)} ei\ d(\omega t)$$

$$= \frac{EI}{\pi}\left[-\cos\omega t\right]_{(\pi/2-\theta/2)}^{(\pi/2+\theta/2)}$$

$$\therefore Mean\ power = \frac{2EI}{\pi}\sin\left(\frac{\theta}{2}\right)\ Watts \quad (1)$$

Note for small θ,

$$Mean\ power = \frac{EI\theta}{\pi}Watts\ , \quad (2)$$

which is intuitively correct.

Calculating power from the fundamental current using Fourier analysis to isolate the fundamental current amplitude (dotted line in Figure 3) yields:

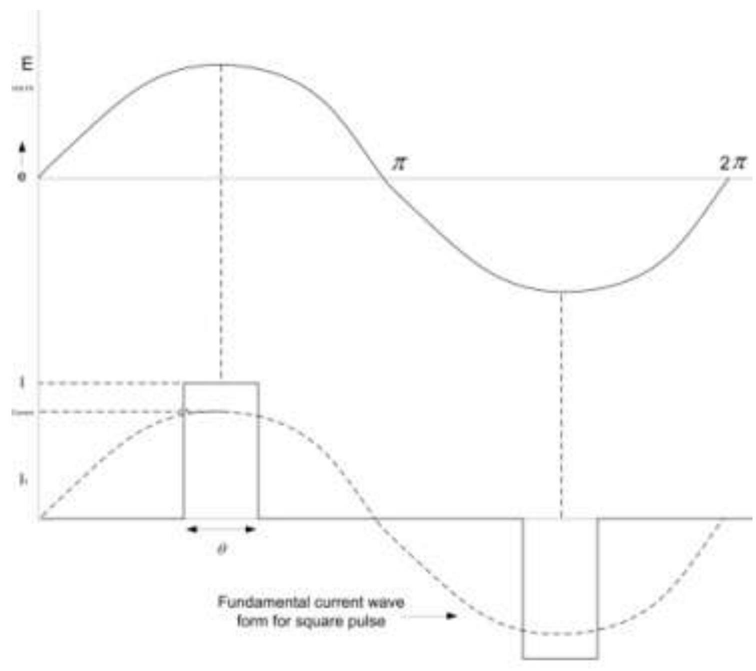

Figure 3

$$I_1 = \frac{I}{\pi}\left[\int_{(\pi/2-\theta/2)}^{(\pi/2+\theta/2)}\sin(\omega t)d\omega t + \int_{(3\pi/2-\theta/2)}^{(3\pi/2+\theta/2)} -\sin(\omega t)d\omega t\right]$$

This reduces to:

$$I_1 = \frac{4I}{\pi}\sin\left(\frac{\theta}{2}\right) \qquad (3)$$

The power in the fundamental frequency, P_1, is given by:

$$P_1 = \frac{E}{\sqrt{2}} \times \frac{I_1}{\sqrt{2}} = \frac{E}{\sqrt{2}} \times \frac{4I\sin\left(\frac{\theta}{2}\right)}{\pi\sqrt{2}}$$

$$\therefore P_1 = \frac{2EI}{\pi}\sin\left(\frac{\theta}{2}\right) Watts$$

This is the same as in equation (1)! It proves that all of the power is from the current fundamental.

Now given that the actual power delivered to the load is proved in two ways above, what about the total power (or VA) delivered to the load? Referring to Figure 1 (or equation 3), note that the rms value, and indeed for any waveform, is given by:

$$I_{rms=}\sqrt{\frac{1}{\pi}\int_{\pi/2-\theta/2}^{\pi/2+\theta/2} I^2 d\omega t}$$

$$\therefore I_{rms} = I\sqrt{\frac{\theta}{\pi}}$$

The total power (or VA) delivered is therefore given by:

$$P_T = \frac{E}{\sqrt{2}} \times I\sqrt{\frac{\theta}{\pi}}$$

The overall Power Factor is then given by Equation 1 divided by Equation 5:

$$P_F = \frac{P_1}{P_T} = \frac{2\frac{E}{\pi}sin\frac{\theta}{2}}{\frac{E}{\sqrt{2}} \times I\sqrt{\frac{\theta}{\pi}}}$$

$$= \frac{2\sqrt{2}}{\sqrt{\pi \cdot \theta}}sin\frac{\theta}{2}$$

Equation 5 is in fact the Distortion Power Factor and explains the difference between the apparent input current (I_{rms}) and the useful input current (I_1). Note that, in this case, I_1 is in phase with E (Figure 3) – this means that the 'normal' power factor or so called Displacement Power Factor is unity. If the fundamental leads by say 10^0 (see Figure 1) then the Displacement Power Factor will be 0.98. The Total Power Factor would then be given by:

$$PF = DPF \times \frac{I_{1rms}}{I_{rms}}$$
$$= 0.98 \times \frac{2\sqrt{2} sin\frac{\theta}{2}}{\sqrt{\pi \cdot \theta}}$$

As an example, for θ = 18^0 or 0.2 radians then the Power Factor (PF) will be:

$$PF = DPF \times 0.62$$

For the waveforms in Figures 1 and 3 the Distortion Power Factor is unity i.e., no phase shift and therefore the Power Factor is 0.62. This means that for every amp entering the load only 0.62 amps is actually used by the load.

An alternative way of looking at Distortion Power Factor is to say that I_{rms} of Equation 7 is actually made up of the rms sum of all the components of the pulse:

$$I_{rms} = \sqrt{\left[\left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_3}{\sqrt{2}}\right)^2 + \left(\frac{I_5}{\sqrt{2}}\right)^2 + \left(\frac{I_7}{\sqrt{2}}\right)^2 + \cdots\cdots\right]}$$

In other words: the actual power is delivered by the fundamental (I_1) and the rest of the power is delivered by the odd harmonics.